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A New Divergence Measure based on Fuzzy TOPSIS for Solving Staff Performance Appraisal

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Abstract

Various divergence measure methods have been used in many applications of fuzzy set theory for calculating the discrimination between two objects. This paper aims to develop a novel divergence measure incorporated with the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method, along with the discussions of its properties. Since ambiguity or uncertainty is an inevitable characteristic of multi-criteria decision-making (MCDM) problems, the fuzzy concept is utilised to convert linguistic expressions into triangular fuzzy numbers. A numerical example of a staff performance appraisal is given to demonstrate suggested method's effectiveness and practicality. Outcomes from this study were compared with various MCDM techniques in terms of correlation coefficients and central processing unit (CPU) time. From the results, there is a slight difference in the ranking order between the proposed method and the other MCDM methods as all the correlation coefficient values are more than 0.9. It is also discovered that CPU time of the proposed method is the lowest compared to the other divergence measure techniques. Hence, the proposed method provides a more sensible and feasible solutions than its counterparts.

Keywords: divergence measure; TOPSIS method; fuzzy concept; linguistic terms; performance appraisal.

1 Introduction

Undoubtedly, performance appraisal is a necessary component of human resource administration in any organisation, private or public. It has recently become an issue for both researchers and practitioners due to globalisation arising from international competition [4, 39]. Performance appraisal has become an indicator of whether or not an organisation performs well and achieves its vision. Methods for assessing staff performance can be either qualitative or quantitative [57]. Organisations should adopt an efficient performance appraisal system that can accurately and fairly evaluate staff performance to channel staff abilities and efforts toward organisational expectations. Managers may take risks by making poor decisions and jeopardising organisational capabilities in the absence of a good performance appraisal system. Consequently, excellent staff may not get an encouraging response and become dissatisfied and quit, causing the organisation to incur excessive hiring costs [32].

Performance appraisal is a method utilised to control and measure previous performance records as planned by the organisation to determine the irregularity of a job to execute corrective action with accurate evidence. It is also used in the workplace to keep employees informed of their work status. The achievement of organisational goals depends greatly on the staff's motivation to perform and their perceptions of the organisation. Perception is defined as the endeavour by which humans arrange and construe their sensory effects to explain their surroundings. Perception varies for different individuals based on their thoughts and experiences. Hence, it is natural for people to have different opinions and arguments [14].

One of the factors contributing to any organisation's achievement is the ability to determine how much the staff completes their tasks in a given period. Organisations that implement the performance appraisal process increase their productivity by about 43% [5]. A study defined performance appraisal as the process of reviewing the work and goals set for the staff by the organisation, the results of which are used to determine their rewards [50]. This is important as it helps with staff training, promotion and transfer, compensation decision, and career development [2]. It has also become an indicator to determine the effectiveness and efficiency of the staff. Hence it must be performed accurately and fairly so that the staff can continuously improve their work performance [2].

There are some basic assumptions made during the execution of a performance appraisal. One of the underlying assumptions is that the contribution of each staff member to the organisation differs from one another due to their different individual performance and that the employers can evaluate and distinguish the staff members [44]. According to Tziner and Kopelman [55], performance appraisal can be cultivated by identifying staff strengths and weaknesses, providing feedback, and interacting with supervisors. A practical performance evaluation device should also consider various components and constraints (e.g., time and costs) in order to formulate and execute such a process [24].

Previous research has found that performance appraisal is very important for the staff in terms of self-definition and short and long-term goal planning, which leads to better work performance [21, 37]. It also has a high potential to improve the organisation's functioning [16]. In this regard, performance appraisal can provide information for pay and promotion decisions, identify development needs and training, and verify selection systems that may qualify for dismissal or sanctions [7]. Performance appraisal, like other decision-making problems, is highly complex because humans struggle to make sound judgments for quantitative problems while making precise predictions for qualitative forecasting. To overcome this issue, the fuzzy linguistic model is used as it can convert verbal expressions into numerical ones [17]. A good performance appraisal process necessitates using an appropriate decision-making method to ensure that the evaluation is implemented effectively and fairly.

The framework for selecting performance appraisal methods and comparing some methods is presented by Jafari et al. [25] to assist the selection process for organisations. This means organisations can evaluate their performance appraisal method according to its key features before implementing any method and incurring additional costs. Anisseh et al. [6] noticed that staff performance appraisal is an element of a group decision-making model in which staff is evaluated according to different points of view. Hence, they presented a fuzzy Delphi method to assess criteria weights and the relative importance of the evaluation group's viewpoints. After that, Andrés et al. [12] proposed a multi-granular linguistic evaluation framework in which the decision-makers could express their assessments on different linguistic scales according to their knowledge of the evaluated staff. Another type of linguistic variable is developed by Dursun and Karsak [13], a 2-tuple linguistic representation model that could manage information assessed using both linguistic and numerical scales in a decision-making problem. With axiomatic support and a presentation of the significance of the parameter α, Gupta et al. [19] apply intuitionistic fuzzy sets for generalisation of fuzzy entropy. This linguistic variable also could help managers to deal with heterogeneous information.

One of the most crucial components used in the operations research field, MCDM is often used to assist evaluators study and choosing the best options via the construction of intricate decision models [54]. Many mathematical programming models have been enhanced to tackle MCDM problems. However, in recent years, the MCDM method has acquired great acceptance for evaluating different proposals [46]. Several of the most regularly used MCDM methods are the analytic hierarchy process (AHP) [47], linear programming technique [18], Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) [23], Preference Ranking Organisation Method for Enrichment Evaluation (PROMETHEE) [48], elimination and choice expressing reality (ELECTRE) [33], simple additive weighting (SAW) [1], etc. Formulated by Hwang and Yoon [23], TOPSIS is a commonly utilised MCDM method because of its advantages in terms of positive and negative ideal solutions and simplicity to implement. Some researchers have applied the classical TOPSIS method to various ambivalent environments. Han and Trimi [20] used fuzzy TOPSIS to select the best reverse logistics company in terms of performance measures. Kumari et al. [30] presented the Shapley-TOPSIS method based on intuitionistic fuzzy sets (IFSs) to determine the foremost options for a cloud service problem. The concept of intuitionistic fuzzy sets in the TOPSIS model also has been extended by Aikhuele et al. [3] for Offshore Wind (OFW) turbine infant failure assessment.

MCDM methods have been used to evaluate, select and rank based on multiple criteria [10, 35]. The MCDM method is a qualitative assessment that emphasises the subjectivity of criteria. It requires information on the criteria chosen and the preference for available criteria [53]. Due to limited resources, MCDM enables decision-makers to determine the aspects among the variables that create the optimal operating environment [1]. This method has been applied in various fields due to its effectiveness in tackling issues related to decision-making. Various recent refinements have been made to improve the method further.

Shannon and Weaver [52] introduced the divergence measure, defined as a measure of discriminatory information. Various entropy measures have been proposed and their properties, and applications were thoroughly discussed [45, 27]. A study provided the axiomatic definition of the divergence measure in the fuzzy environment and computed discrimination for fuzzy sets. In other words, the divergence measure describes the dissimilarity and several interesting axioms for approximating fuzzy set discrimination [36]. Meanwhile, Hung and Yang [22] developed the J-divergence measure between IFSs. The significance of appropriate distance measures between

IFSs emerges due to their role in the inference problem. The proposed divergence measure can calculate practical distances and measures of similarity between IFSs. Ghosh et al. [15] came up with a method to calculate fuzzy divergence in the field of automated leukocyte recognition. Recently, Parkash and Kumar [40] presented a modified fuzzy divergence measure to remove the drawbacks of previous divergence measures in literature and discuss its detailed properties. Afterwards, Joshi and Kumar [26] introduced a divergence measure that was established by well-known Shannon entropy concept. Additionally, some properties of the proposed divergence measure are also discussed. Then, Rani et al. [41] proposed a method based on divergence measure for fuzzy sets (FSs) to evaluate the MCDM problems under the fuzzy atmosphere. After a short period, Rani et al. [42] proposed a fuzzy TOPSIS method with a divergence measure to tackle decision-making issues. However, some of the previous divergence measures are limited because they can only be used after the defuzzification process has been completed. If the defuzzification process is neglected, the divergence measures could not evaluate the value in a fuzzy interval whenever the value is 0 or 1.

Hence, this study aims to propose a novel divergence measure that can overcome this limitation, and to evaluate any possible score value of alternatives to resolve any issues with the current measures, including any result abnormalities. Furthermore, a decision-making problem of selecting the best candidate for staff performance appraisal is considered to exhibit the comprehensive implementation process of the developed technique. The main contributions of this study are as follows:

1. Proposing a new divergence measure that can remove the shortcoming of previous divergence measures apart from reviewing and presenting some existing measures as in literature.
2. Enhancing the standard TOPSIS method under FSs to solve the MCDM problems of selecting the best candidate for staff performance appraisal.
3. Constructing an aggregation connective measure for each element of the decision matrix to evaluate overall performance for each alternative.
4. Introducing an MCDM problem of the staff performance appraisal to explore the extent of its validity and applicability of fuzzy TOPSIS under the proposed divergence measure.
5. Finally, make comparisons, evaluating the correlation coefficient and CPU time to verify the proposed method.

The remaining of this paper will be organised as follows: Section 2 presents the preliminaries that include the proposed divergence measure. Section 3 discusses the algorithms for the improved fuzzy TOPSIS method. Section 4 gives a sample of performance appraisal using the improved fuzzy TOPSIS, demonstrating the effectiveness of the method suggested in this study. Next, Section 5 will discuss and compare between the proposed study with the current MCDM methods, while the overall conclusion is given in Section 6.

2 Preliminaries

This section discusses some fundamental concepts of fuzzy set theories and the new divergence measure.

2.1 Fuzzy Sets

Definition 1 [60]

Let $Y = \{y_1, y_2, ..., y_n\}$ be a finite discourse set. A fuzzy set M defined on Y is described by a membership function $\mu_M(y_i)$ and given as:

$$M = \{(y_i, \mu_M(y_i)) : \mu_M(y_i) \in [0,1]; \forall y_i \in Y\}, \tag{1}$$

where the function value $\mu_M(y_i)$ is called the membership degree of y_i to M in Y.

Definition 2 [8, 60]

A triangular fuzzy number (TFN) is represented by $\tilde{a} = (a_1, a_2, a_3)$ and depicted in Figure 1. The membership function $\mu_{\tilde{a}}(y)$ of TFN $\tilde{a}$ is defined as follows:

$$\mu_{\tilde{a}}(y) = \begin{cases} 0 & \text{if } y < a_1, \\ \dfrac{y - a_1}{a_2 - a_1} & \text{if } a_1 \leq y < a_2, \\ \dfrac{a_3 - y}{a_3 - a_2} & \text{if } a_2 \leq y \leq a_3, \\ 0 & \text{if } y > a_3, \end{cases} \tag{2}$$

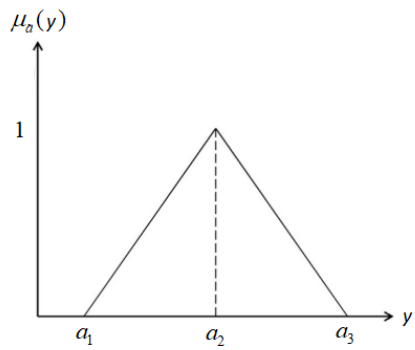

Figure 1: Triangular fuzzy number with membership function.

Definition 3 [38]

The arithmetic operations of TFN, namely $\tilde{\Gamma} = (\gamma_1, \gamma_2, \gamma_3)$ and $\tilde{\Delta} = (\delta_1, \delta_2, \delta_3)$, with k as a positive real number, are defined as follows:

1. Addition: $\tilde{\Gamma}(+)\tilde{\Delta} = (\gamma_1 + \delta_1, \gamma_2 + \delta_2, \gamma_3 + \delta_3)$.
2. Subtraction: $\tilde{\Gamma}(-)\tilde{\Delta} = (\gamma_1 - \delta_1, \gamma_2 - \delta_2, \gamma_3 - \delta_3)$.
3. Multiplication: $\tilde{\Gamma}(\times)\tilde{\Delta} = (\min(\gamma_1\delta_1, \gamma_1\delta_3, \gamma_3\delta_1, \gamma_3\delta_3), \gamma_2\delta_2, \max(\gamma_1\delta_1, \gamma_1\delta_3, \gamma_3\delta_1, \gamma_3\delta_3)), c(\times)\Gamma = (c \times \gamma_1, c \times \gamma_2, c \times \gamma_3)$.
4. Division: $\tilde{\Gamma}(\div)\tilde{\Delta} = (\min(\gamma_1/\delta_1, \gamma_1/\delta_3, \gamma_3/\delta_1, \gamma_3/\delta_3), \gamma_2/\delta_2, \max(\gamma_1/\delta_1, \gamma_1/\delta_3, \gamma_3/\delta_1, \gamma_3/\delta_3))$

Definition 4 [31, 61]

The graded mean integration representation value, $R(\tilde{a})$ of TFN $\tilde{a} = (l, m, u)$ is defined as follows:

$$R(\tilde{a}) = \frac{l + 4m + u}{6}. \tag{3}$$

2.2 New Divergence Measure in Fuzzy Sets

The degree of discrimination between fuzzy sets can be determined using the divergence measure. It is a significant approach that has been applied in various fields, such as medical diagnosis [59], image segmentation [56], pattern recognition [58] and decision-making [43].

In the theory of information, the concept of divergence measure was first of all introduced by Shannon and Weaver [52] which is defined as follows:

Let $\Delta_n = \{A = (a_1, a_2, ..., a_n) : a_i \geq 0, i = 1, 2, ..., n; \sum_{i=1}^{n} a_i = 1\}, n \geq 2$ be set of n-complete probability distributions. For any probability distribution $A = (a_1, a_2, ..., a_n) \in \Delta_n$, an entropy is defined as:

$$H(A) = -\sum_{i=1}^{n} a_i \log(a_i). \tag{4}$$

After that, Kullback and Leibler [29] generalized this concept to find the divergence measure of $A = (a_1, a_2, ..., a_n) \in \Delta_n$ from $B = (b_1, b_2, ..., b_n) \in \Delta_n$ as:

$$KL(A : B) = \sum_{i=1}^{n} a_i \log\left(\frac{a_i}{b_i}\right). \tag{5}$$

Then, Kullback [28] have got the idea to propose the symmetric divergence measure as:

$$K(A : B) = KL(A : B) + KL(B : A) = \sum_{i=1}^{n} (a_i - b_i) \log\left(\frac{a_i}{b_i}\right). \tag{6}$$

Inspired by this, Bhandari et al. [9] suggested the following fuzzy divergence measure of fuzzy set $M \in FS(X)$ from $N \in FS(X)$ as:

$$I(M, N) = \sum_{i=1}^{n} \left[\mu_M(x_i) \log\left(\frac{\mu_M(x_i)}{\mu_N(x_i)}\right) + (1 - \mu_M(x_i)) \log\left(\frac{1 - \mu_M(x_i)}{1 - \mu_N(x_i)}\right)\right], \tag{7}$$

and the respective symmetric divergence measure by

$$J(M, N) = I(M, N) + I(N, M),$$

which can be simplified as:

$$J(M, N) = \sum_{i=1}^{n} \left[(\mu_M(x_i) - \mu_N(x_i)) \log\left(\frac{\mu_M(x_i)(1 - \mu_N(x_i))}{\mu_N(x_i)(1 - \mu_M(x_i))}\right)\right]. \tag{8}$$

Recently, Rani et al. [42] proposed a new divergence measure of fuzzy sets which is defined as:

$$J(P,Q) = \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\mu_P(x_i)+\mu_Q(x_i)}{2}\right)\ln\left(\frac{\mu_P(x_i)+\mu_Q(x_i)}{2\mu_P(x_i)}\right) + \left(\frac{2-\mu_P(x_i)-\mu_Q(x_i)}{2}\right) \ln\left(\frac{2-\mu_P(x_i)-\mu_Q(x_i)}{2(1-\mu_P(x_i))}\right) + \left(\frac{\mu_P(x_i)+\mu_Q(x_i)}{2}\right)\ln\left(\frac{\mu_P(x_i)+\mu_Q(x_i)}{2\mu_Q(x_i)}\right) + \left(\frac{2-\mu_P(x_i)-\mu_Q(x_i)}{2}\right)\ln\left(\frac{2-\mu_P(x_i)-\mu_Q(x_i)}{2(1-\mu_Q(x_i))}\right), \tag{9}$$

where $P \in FS(X)$ and $Q \in FS(X)$. Although the divergence measure is used to determine discrimination between fuzzy sets, in this case, it has a limitation in which it only can be used after the defuzzification process since the fuzzy number is in interval form.

Therefore, in this paper, we propose a new divergence measure that can overcome this limitation and evaluate any possible score value of alternatives that overcomes the deficiency of the existing measures and eliminate the abnormality of results. In relation to the divergence measure presented in (9), this study aims to propose a new divergence measure of fuzzy set $Y \in FS(X)$ from $Z \in FS(X)$ as per below:

$$J(Y,Z) = \sum_{i=1}^{n}(\mu_Y(x_i)-\mu_Z(x_i))\ln\left(\frac{\mu_Y(x_i)+1}{\mu_Z(x_i)+1}\right) + (\mu_Z(x_i)-\mu_Y(x_i))\ln\left(\frac{2-\mu_Y(x_i)}{2-\mu_Z(x_i)}\right). \tag{10}$$

The proposed divergence measure satisfies some properties as the following theorems.

Theorem 2.1. *Let $Y, Z \in FSs(X)$, then divergence measure $J(Y,Z)$ defined in Equation (10) holds the following properties, that are given as follows:*

1. $J(Y,Z) \geq 0$,
2. $J(Y,Z) = 0$ *if* $Y = Z$,
3. $J(Y,Z) = J(Z,Y)$.

Proof. (1) and (2): Let

$$J(Y,Z) = \sum_{i=1}^{n} f(\mu_Y(x_i),\mu_Z(x_i)),$$

and

$$f(\mu_Y(x_i),\mu_Z(x_i)) = (\mu_Y(x_i)-\mu_Z(x_i))\ln\left(\frac{\mu_Y(x_i)+1}{\mu_Z(x_i)+1}\right) + (\mu_Z(x_i)-\mu_Y(x_i))\ln\left(\frac{2-\mu_Y(x_i)}{2-\mu_Z(x_i)}\right),$$

then the first partial derivatives of f with respect to $\mu_Y(x_i)$ is given by:

$$f_{\mu_Y(x_i)} = \frac{\mu_Y(x_i)-\mu_Z(x_i)}{\mu_Y(x_i)+1} - \frac{\mu_Z(x_i)-\mu_Y(x_i)}{2-\mu_Y(x_i)} + \ln\left(\frac{\mu_Y(x_i)+1}{\mu_Z(x_i)+1}\right) - \ln\left(\frac{2-\mu_Y(x_i)}{2-\mu_Z(x_i)}\right),$$

and also, the second partial derivatives of f with respect to $\mu_Y(x_i)$ is given by:

$$f_{\mu_Y(x_i)\mu_Y(x_i)} = \frac{2+\mu_Y(x_i)+\mu_Z(x_i)}{(\mu_Y(x_i)+1)^2} + \frac{4-\mu_Y(x_i)-\mu_Z(x_i)}{(2-\mu_Y(x_i))^2}.$$

Since $f_{\mu_Y(x_i)\mu_Y(x_i)} \geq 0$ for $\mu_Y(x_i), \mu_Z(x_i) \in [0,1]$, thus f is a concave up mapping of $\mu_Y(x_i)$ and $J(Y,Z)$ is a convex function. With constant $\mu_Z(x_i) \in [0,1]$, $f(\mu_Y(x_i), \mu_Z(x_i))$ is decreasing in $[\mu_Y(x_i), \mu_Z(x_i)]$ and increasing in $[\mu_Z(x_i), \mu_Y(x_i)]$. Therefore, when $\mu_Y(x_i) \in [0, \mu_Z(x_i)]$, $f(\mu_Z(x_i), \mu_Z(x_i)) \leq f(\mu_Y(x_i), \mu_Z(x_i)) \leq f(0, \mu_Z(x_i))$ and similarly for $\mu_Y(x_i) \in [\mu_Z(x_i), 1]$, $f(\mu_Z(x_i), \mu_Z(x_i)) \leq f(\mu_Y(x_i), \mu_Z(x_i)) \leq f(1, \mu_Z(x_i))$. Hence for $\mu_Y(x_i) \in [0,1]$ with constant $\mu_Z(x_i) \in [0,1]$, $f(\mu_Y(x_i), \mu_Z(x_i))$ attains its maximum at $\mu_Y(x_i) = \{1\}$, $\mu_Z(x_i) = \{0\}$ (or $\mu_Y(x_i) = \{0\}$, $\mu_Z(x_i) = \{1\}$) and its minimum at $\mu_Y(x_i) = \mu_Z(x_i)$.

Hence, $0 \leq J(X,Y) \leq \ln(4)$ and $J(X,Y) = 0$ if $\mu_Y(x_i) = \mu_Z(x_i)$.

(3): Suppose

$$\begin{aligned}
J(Y,Z) &= \sum_{i=1}^{n} (\mu_Y(x_i) - \mu_Z(x_i)) \ln\left(\frac{\mu_Y(x_i)+1}{\mu_Z(x_i)+1}\right) + (\mu_Z(x_i) - \mu_Y(x_i)) \ln\left(\frac{2-\mu_Y(x_i)}{2-\mu_Z(x_i)}\right) \\
&= \sum_{i=1}^{n} -(\mu_Z(x_i) - \mu_Y(x_i)) \ln\left(\frac{\mu_Z(x_i)+1}{\mu_Y(x_i)+1}\right)^{-1} - (\mu_Y(x_i) - \mu_Z(x_i)) \ln\left(\frac{2-\mu_Z(x_i)}{2-\mu_Y(x_i)}\right)^{-1} \\
&= \sum_{i=1}^{n} (\mu_Z(x_i) - \mu_Y(x_i)) \ln\left(\frac{\mu_Z(x_i)+1}{\mu_Y(x_i)+1}\right) + (\mu_Y(x_i) - \mu_Z(x_i)) \ln\left(\frac{2-\mu_Z(x_i)}{2-\mu_Y(x_i)}\right) \\
&= J(Z,Y).
\end{aligned}$$

Hence, it is proven that $J(Y,Z) = J(Z,Y)$. □

Theorem 2.2. *For $Y, Z, P \in FS(X)$,*

1. $J(Y \cup Z, Y \cap Z) = J(Y,Z)$,
2. $J(Y \cup Z, Y) + J(Y \cap Z, Y) = J(Y,Z)$,
3. $J(Y \cup Z, P) \leq J(Y,P) + J(Z,P)$,
4. $J(Y \cap Z, P) \leq J(Y,P) + J(Z,P)$.

Proof. Let $X_1 = \{x_i \in X, \mu_Y(x_i) \leq \mu_Z(x_i)\}$, then
$Y \cup Z =$ Union of Y and $Z \Leftrightarrow \mu_{Y\cup Z}(x_i) = \max\{\mu_Y(x_i), \mu_Z(x_i)\} = \mu_Z(x_i)$;
$Y \cap Z =$ Intersection of Y and $Z \Leftrightarrow \mu_{Y\cap Z}(x_i) = \min\{\mu_Y(x_i), \mu_Z(x_i)\} = \mu_Y(x_i)$.

(1): Suppose

$$\begin{aligned}
J(Y\cup Z, Y\cap Z) = &\sum_{i=1}^{n} (\mu_{Y\cup Z}(x_i) - \mu_{Y\cap Z}(x_i)) \ln\left(\frac{\mu_{Y\cup Z}(x_i)+1}{\mu_{Y\cap Z}(x_i)+1}\right) \\
&+ (\mu_{Y\cap Z}(x_i) - \mu_{Y\cup Z}(x_i)) \ln\left(\frac{2-\mu_{Y\cup Z}(x_i)}{2-\mu_{Y\cap Z}(x_i)}\right) \\
=&\sum_{i=1}^{n} (\mu_Z(x_i) - \mu_Y(x_i)) \ln\left(\frac{\mu_Z(x_i)+1}{\mu_Y(x_i)+1}\right) + (\mu_Y(x_i) - \mu_Z(x_i)) \ln\left(\frac{2-\mu_Z(x_i)}{2-\mu_Y(x_i)}\right) \\
=&J(Z,Y).
\end{aligned}$$

By using property (3) in Theorem 2.1, $J(Z,Y) = J(Y,Z)$. Hence, $J(Y\cup Z, Y\cap Z) = J(Y,Z)$.

(2) can similarly be proved as (1).

(3): Suppose

$$\begin{aligned} J(Y\cup Z,P) =& \sum_{i=1}^{n}(\mu_{Y\cup Z}(x_i)-\mu_P(x_i))\ln\left(\frac{\mu_{Y\cup Z}(x_i)+1}{\mu_P(x_i)+1}\right) \\ &+(\mu_P(x_i)-\mu_{Y\cup Z}(x_i))\ln\left(\frac{2-\mu_{Y\cup Z}(x_i)}{2-\mu_P(x_i)}\right) \\ =&\sum_{i=1}^{n}(\mu_Z(x_i)-\mu_P(x_i))\ln\left(\frac{\mu_Z(x_i)+1}{\mu_P(x_i)+1}\right)+(\mu_P(x_i)-\mu_Z(x_i))\ln\left(\frac{2-\mu_Z(x_i)}{2-\mu_P(x_i)}\right) \\ =&J(Z,P). \end{aligned}$$

Since $J(Y,P)\geq 0$ as in Theorem 2.1, then $J(Y\cup Z,P)\leq J(Y,P)+J(Z,P)$.

(4) can similarly be proved as (3). $\square$

3 Improved Fuzzy TOPSIS Approach

This section presents an improved fuzzy TOPSIS approach that is beneficial in tackling MCDM issues using the new divergence measure.

Assume the MCDM issue has m alternatives, $A=\{A_1,A_2,...,A_m\}$ and the alternatives are appraised using n criteria, $C=\{C_1,C_2,...,C_n\}$ and sub-criteria, $S=\{C_{11},C_{12},...,C_{np}\}$ where p denotes the number of sub-criteria in the main criteria n. Let $\phi_j(j=1(1)n)$ be the weight of the main criterion and $w^*_{jk}(j=1(1)n,k=1(1)p)$ be the weight of the sub-criterion, such that $\phi_j\geq 0$, $w^*_{jk}\geq 0$, $\sum_{j=1}^{n}\phi_j=1$ and $\sum_{k=1}^{p}w^*_{jk}=1(j=1(1)n)$. All criteria an alternatives are evaluated by several experts $E=\{E_1,E_2,...,E_l\}$ based on linguistic terms. The proposed method includes several steps, as follows:

Step 1: Develop a fuzzy decision matrix $\tilde{F}=\left(\tilde{\xi}^{(u)}_{ijk}\right)_{m\times n}$ and weighting matrix $\tilde{W}=\left(\tilde{w}^{(u)}_{jk}\right)_{l\times(a+b+p)}$.

The experts provide the feasible assessments of alternative A_i regarding criterion C_j and sub-criterion C_{jk} represented by the fuzzy numbers $\tilde{\xi}^{(u)}_{ijk}=(f_{ijk},g_{ijk},h_{ijk})$ acquired from linguistic variables in Table 1 and demonstrated as:

$$\tilde{F}=\begin{array}{c} \\ \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{ccccccccccccc} & C_1 & & & & C_2 & & & \cdots & & C_n & & \\ C_{11} & C_{12} & \cdots & C_{1a} & C_{21} & C_{22} & \cdots & C_{2b} & \cdots & C_{n1} & C_{n2} & \cdots & C_{np} \end{array} \\ \begin{pmatrix} \tilde{\xi}^{(u)}_{111} & \tilde{\xi}^{(u)}_{112} & \cdots & \tilde{\xi}^{(u)}_{11a} & \tilde{\xi}^{(u)}_{121} & \tilde{\xi}^{(u)}_{122} & \cdots & \tilde{\xi}^{(u)}_{12b} & \cdots & \tilde{\xi}^{(u)}_{1n1} & \tilde{\xi}^{(u)}_{1n2} & \cdots & \tilde{\xi}^{(u)}_{1np} \\ \tilde{\xi}^{(u)}_{211} & \tilde{\xi}^{(u)}_{212} & \cdots & \tilde{\xi}^{(u)}_{21a} & \tilde{\xi}^{(u)}_{221} & \tilde{\xi}^{(u)}_{222} & \cdots & \tilde{\xi}^{(u)}_{22b} & \cdots & \tilde{\xi}^{(u)}_{2n1} & \tilde{\xi}^{(u)}_{2n2} & \cdots & \tilde{\xi}^{(u)}_{2np} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\xi}^{(u)}_{m11} & \tilde{\xi}^{(u)}_{m12} & \cdots & \tilde{\xi}^{(u)}_{m1a} & \tilde{\xi}^{(u)}_{m21} & \tilde{\xi}^{(u)}_{m22} & \cdots & \tilde{\xi}^{(u)}_{m2b} & \cdots & \tilde{\xi}^{(u)}_{mn1} & \tilde{\xi}^{(u)}_{mn2} & \cdots & \tilde{\xi}^{(u)}_{mnp} \end{pmatrix} \end{array} \tag{11}$$

for $u = 1, 2, ..., l$.

Table 1: Linguistic terms that represent fuzzy numbers used for evaluating each alternative.

Linguistics Terms	Fuzzy Numbers
Terrible (λ)	$(0, 0, 1)$
Medium Terrible (κ)	$(0, 1, 2)$
Very Poor (ι)	$(1, 2, 3)$
Poor (θ)	$(2, 3, 4)$
Medium Fair (η)	$(3, 4, 5)$
Fair (ζ)	$(4, 5, 6)$
Medium Good (ϵ)	$(5, 6, 7)$
Good (δ)	$(6, 7, 8)$
Very Good (γ)	$(7, 8, 9)$
Medium Excellent (β)	$(8, 9, 10)$
Excellent (α)	$(9, 10, 10)$

The experts also assess the weights of main and sub-criteria represented by the fuzzy numbers $\tilde{\phi}_j^{(u)} = (f_j, g_j, h_j)$ and $\tilde{w}_{jk}^{(u)} = (f_{jk}, g_{jk}, h_{jk})$ based on Table 2 and demonstrated respectively as:

$$\tilde{\Phi} = \begin{array}{c} \\ E_1 \\ E_2 \\ \vdots \\ E_l \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \tilde{\phi}_1^{(1)} & \tilde{\phi}_2^{(1)} & \cdots & \tilde{\phi}_n^{(1)} \\ \tilde{\phi}_1^{(2)} & \tilde{\phi}_2^{(2)} & \cdots & \tilde{\phi}_n^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\phi}_1^{(l)} & \tilde{\phi}_2^{(l)} & \cdots & \tilde{\phi}_n^{(l)} \end{pmatrix} \end{array} \tag{12}$$

and

$$\tilde{W} = \begin{array}{c} \\ \\ E_1 \\ E_2 \\ \vdots \\ E_l \end{array} \begin{array}{c} \begin{array}{ccccccccccccc} & C_1 & & & & C_2 & & & & & C_n & & \\ C_{11} & C_{12} & \cdots & C_{1a} & C_{21} & C_{22} & \cdots & C_{2b} & \cdots & C_{n1} & C_{n2} & \cdots & C_{np} \end{array} \\ \begin{pmatrix} \tilde{w}_{11}^{(1)} & \tilde{w}_{12}^{(1)} & \cdots & \tilde{w}_{1a}^{(1)} & \tilde{w}_{21}^{(1)} & \tilde{w}_{22}^{(1)} & \cdots & \tilde{w}_{2b}^{(1)} & \cdots & \tilde{w}_{n1}^{(1)} & \tilde{w}_{n2}^{(1)} & \cdots & \tilde{w}_{np}^{(1)} \\ \tilde{w}_{11}^{(2)} & \tilde{w}_{12}^{(2)} & \cdots & \tilde{w}_{1a}^{(2)} & \tilde{w}_{21}^{(2)} & \tilde{w}_{22}^{(2)} & \cdots & \tilde{w}_{2b}^{(2)} & \cdots & \tilde{w}_{n1}^{(2)} & \tilde{w}_{n2}^{(2)} & \cdots & \tilde{w}_{np}^{(2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \tilde{w}_{11}^{(l)} & \tilde{w}_{12}^{(l)} & \cdots & \tilde{w}_{1a}^{(l)} & \tilde{w}_{21}^{(l)} & \tilde{w}_{22}^{(l)} & \cdots & \tilde{w}_{2b}^{(l)} & \cdots & \tilde{w}_{n1}^{(l)} & \tilde{w}_{n2}^{(l)} & \cdots & \tilde{w}_{np}^{(l)} \end{pmatrix} \end{array} \tag{13}$$

Table 2: Linguistic terms that represent fuzzy numbers used for evaluating each criterion.

Linguistics Terms	Fuzzy Numbers
Very Low (VL)	$(0, 0, 0.2)$
Low(L)	$(0.05, 0.2, 0.35)$
Medium Low (ML)	$(0.2, 0.35, 0.5)$
Medium (M)	$(0.35, 0.5, 0.65)$
Medium High (MH)	$(0.5, 0.65, 0.8)$
High (H)	$(0.65, 0.8, 0.95)$
Very High (VH)	$(0.8, 1, 1)$

Step 2: Aggregate the fuzzy evaluations of alternatives and the fuzzy weights of criteria via the equations provided:

$$\tilde{\xi}_{ijk} = \frac{1}{l}\left[\tilde{\xi}^{(1)}_{ijk}(+)\tilde{\xi}^{(2)}_{ijk}(+)...(+)\tilde{\xi}^{(l)}_{ijk}\right], \tag{14}$$

$$\tilde{\phi}_{ijk} = \frac{1}{l}\left[\tilde{\phi}^{(1)}_{ijk}(+)\tilde{\phi}^{(2)}_{ijk}(+)...(+)\tilde{\phi}^{(l)}_{ijk}\right], \tag{15}$$

$$\tilde{w}_{ijk} = \frac{1}{l}\left[\tilde{w}^{(1)}_{ijk}(+)\tilde{w}^{(2)}_{ijk}(+)...(+)\tilde{w}^{(l)}_{ijk}\right]. \tag{16}$$

In this study, it is worth mentioning that the preference of each expert is assumed to be equal since they have an equal level of knowledge.

Step 3: Normalise the fuzzy decision matrix.

Normalisation aims to eliminate the difference between the attributes in magnitude and dimension, in which the normalised value is in the range of [0, 1]. Hence, the technical problems generated by distinct measurement categories can be eliminated [51, 11]. The preliminary data corresponding to each criterion is normalised by dividing it by the most dominant criterion value. The element of a normalised decision matrix $\tilde{r}_{ijk}$ resulting from TFN $\tilde{\xi}^{(u)}_{ijk} = (f_{ijk}, g_{ijk}, h_{ijk})$ is given by [20]:

$$\tilde{r}_{ijk} = \left(\frac{f_{ijk}}{h^{\max}_{jk}}, \frac{g_{ijk}}{h^{\max}_{jk}}, \frac{h_{ijk}}{h^{\max}_{jk}}\right), i = 1(1)m; j = 1(1)n; k = 1(1)p, \text{ for benefit criteria, and} \tag{17}$$

$$\tilde{r}_{ijk} = \left(\frac{f^{\min}_{jk}}{h_{ijk}}, \frac{f^{\min}_{jk}}{g_{ijk}}, \frac{f^{\min}_{jk}}{f_{ijk}}\right), i = 1(1)m; j = 1(1)n; k = 1(1)p, \text{ for cost criteria.} \tag{18}$$

Step 4: Defuzzify the fuzzy decision matrix and weighting matrix.

The element of the decision matrix is in the TFN form. The fuzzy numbers must be properly defuzzified to crisp values to execute the model. Defuzzification is a method to transform fuzzy values back into crisp values. Different defuzzification methods yield different formulas or approaches, which result in different defuzzified values that may aid in obtaining different ranking outcomes [34, 49]. Some available defuzzification methods include the centroid method, graded mean integration representation (GMIR), center of mass, and mean of maxima [61, 49]. In this study, the crisp value Crisp($\tilde{a}$) for TFN $\tilde{a} = (a_1, a_2, a_3)$ was determined using the GMIR method as in Equation (3).

Step 5: Define the positive-ideal solution (PIS) and negative-ideal solution (NIS) with regard to the decision matrix's defuzzified values.

The PIS (Z^+) and NIS (Z^-) are as follow:

$$Z^+ = \{\zeta_{11}^+, \zeta_{12}^+, ..., \zeta_{np}^+\}, \text{ where } \zeta_{jk}^+ = \max_i\{r_{ijk}\}, i = 1(1)m; j = 1(1)n; k = 1(1)p, \text{ and} \tag{19}$$

$$Z^- = \{\zeta_{11}^-, \zeta_{12}^-, ..., \zeta_{np}^-\}, \text{ where } \zeta_{jk}^- = \min_i\{r_{ijk}\}, i = 1(1)m; j = 1(1)n; k = 1(1)p. \tag{20}$$

Step 6: Compute the separation measures d_{ijk}^+ and d_{ijk}^- of defuzzified values r_{ijk} from PIS and NIS respectively, using the proposed divergence measures.

$$d_{ijk}^+ = (r_{ijk} - \zeta_{jk}^+) \ln\left(\frac{r_{ijk}+1}{\zeta_{jk}^+ + 1}\right) + (\zeta_{jk}^+ - r_{ijk}) \ln\left(\frac{2-r_{ijk}}{2-\zeta_{jk}^+}\right), \text{ and} \tag{21}$$

$$d_{ijk}^- = (r_{ijk} - \zeta_{jk}^-) \ln\left(\frac{r_{ijk}+1}{\zeta_{jk}^- + 1}\right) + (\zeta_{jk}^- - r_{ijk}) \ln\left(\frac{2-r_{ijk}}{2-\zeta_{jk}^-}\right). \tag{22}$$

Step 7: Compute the functions of μ_{ijk} and $\nu_(ijk)$ regarding the separation measures.

$$\mu_{ijk} = \frac{1}{1 + d_{ijk}^+}, \tag{23}$$

such that if $d_{ijk}^+ = 0$, then $\mu_{ijk} = 1$ and if $d_{ijk}^+ \to \infty$, then $\mu_{ijk} = 0$ and

$$\nu_{ijk} = 1 - \frac{1}{1 + d_{ijk}^-} = \frac{d_{ijk}^-}{1 + d_{ijk}^-}, \tag{24}$$

such that if $d_{ijk}^- = 0$, then $\nu_{ijk} = 0$ and if $d_{ijk}^- \to \infty$, then $\nu_{ijk} = 1$.

Step 8: Compute the aggregation connective measure ρ_{ijk} for the functions of $\mu_{ijk} = 1$ and $\nu_{ijk} = 1$.

$$\rho_{ijk} = \mu_{ijk} \cap_p \nu_{ijk} = 1 - \min\left[1, [(1-\mu_{ijk})^p + (1-\nu_{ijk})^p]^{\frac{1}{p}}\right], \text{ for } p \geq 1. \tag{25}$$

Step 9: Compute the weights of sub-criteria. The sub-criteria weights for each primary criterion should be normalised to correlate to each element of the defuzzified weighting matrix, where $\sum_{k=1}^{p} = 1 (j = 1(1)n)$. The weight of each sub-criteria is defined as follows:

$$w_{jk}^* = \frac{w_{jk}}{\sum_{k=1}^{p} w_{jk}} \text{ for } j = 1(1)n. \tag{26}$$

Step 10: Compute the overall performance of the alternatives.

$$J(A_i) = \phi_1 \sum_{k=1}^{a} w_{1k}^* \rho_{i1k} + \phi_2 \sum_{k=1}^{b} w_{2k}^* \rho_{i2k} + ... + \phi_n \sum_{k=1}^{p} w_{nk}^* \rho_{ink} \tag{27}$$

Step 11: Rank the alternatives. Sort the alternatives from top to bottom based on their performance, with the greatest value being the best option.

4 Numerical Application

To test and confirm the efficiency of the method suggested in this study, the method will be used on a Malaysian university's staff performance appraisal case study.

Let $A = \{A_1, A_2, ..., A_{15}\}$ be the selected candidates from the university that are evaluated based on four main criteria $C = \{C_1, C_2, C_3, C_4\}$ and 14 sub-criteria $S = \{C_{11}, C_{12}, ..., C_{41}\}$. An expert group $E = \{E_1, E_2\}$ was established in order to assess the alternatives and criteria weights from linguistic viewpoint. The definition of main criteria, sub-criteria and alternatives involved in this assessment are provided in Table 3. All candidates were measured and ranked using the proposed method based on the established criteria.

Table 3: Definition of selected alternatives and criteria.

Main Criteria	Sub-criteria	Alternatives
Work Execution (C_1)	Quantity of work (C_{11})	Candidate A_1
	Quality of work regarding perfection and neatness (C_{12})	Candidate A_2 Candidate A_3
	Quality of work regarding efforts and initiatives to attain work perfection (C_{13})	Candidate A_4 Candidate A_5
	Time management (C_{14})	Candidate A_6
	Work efficacy (C_{15})	Candidate A_7
Knowledge and Expertise (C_2)	Knowledge and expertise in the field of works (C_{21})	Candidate A_8 Candidate A_9
	Execution of policies, regulation and administrative order (C_{22})	Candidate A_{10} Candidate A_{11}
	The efficacy of communication (C_{23})	Candidate A_{12}
Personal Attributes (C_3)	Leadership skills (C_{31})	Candidate A_{13}
	Ability to organise (C_{32})	Candidate A_{14}
	Discipline (C_{33})	Candidate A_{15}
	Proactive and innovative (C_{34})	
	Connection and collaboration (C_{35})	
Contributions other than Office Duties (C_4)	Activities involvement in any level (C_{41})	

Step 1: The fuzzy decision matrix and weighting matrix were developed from the linguistic variables listed in Tables 1 and 2, respectively. The linguistic variables represent TFNs that were used by experts to review the alternatives and criteria weights. The performance ratings of the alternatives by the two experts are given in Appendix A. The evaluation of main and sub-criteria weights by the two experts is shown in Tables 4 and 5, respectively.

Table 4: Expert evaluation of each main criterion weight.

Main criteria	Weight
C_1	0.50
C_2	0.25
C_3	0.20
C_4	0.05

Table 5: Expert evaluation of each sub-criterion weight.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
E_1	H	VH	MH	VH	H	H	VH	MH	MH	H	VH	H	MH	M
E_2	MH	H	VH	H	VH	VH	H	H	MH	H	VH	H	H	MH

Table 6: Weights of sub-criteria using the GMIR procedure.

Main Criteria	Sub-criteria	Weight, w^*_{jk}
C_1	C_{11}	0.1733
	C_{12}	0.2112
	C_{13}	0.1932
	C_{14}	0.2112
	C_{15}	0.2112
C_2	C_{21}	0.3545
	C_{22}	0.3545
	C_{23}	0.2910
C_3	C_{31}	0.1649
	C_{32}	0.2030
	C_{33}	0.2452
	C_{34}	0.2030
	C_{35}	0.1839
C_4	C_{41}	1.0000

Step 2: The fuzzy evaluations of alternatives and the fuzzy weights of criteria were aggregated using Equations (14) and (16), respectively.

Step 3: The normalised fuzzy decision matrix was constructed based on Equations (17) and (18).

Step 4: Defuzzification of the fuzzy decision matrix and weighting matrix was implemented using Equation (3).

Step 5: The following are the PIS and NIS:

$$Z^+ = \{0.983, 0.9, 0.9, 0.9, 0.9, 0.983, 0.983, 0.983, 0.983, 0.983, 0.983, 0.983, 0.983, 0.9\},$$
$$Z^- = \{0.9, 0.8, 0.8, 0.8, 0.8, 0.9, 0.9, 0.8, 0.8, 0.8, 0.983, 0.8, 0.8, 0.8\}.$$

Step 6: The separation measure d^+_{ijk} and d^-_{ijk} of defuzzified values r_{ijk} were computed using Equations (21) and (22), respectively.

Step 7: The functions of μ_{ijk} and ν_{ijk} were calculated using Equations (23) and (24), respectively.

Step 8: The aggregation connective measure for the functions of μ_{ijk} and ν_{ijk} was defined using Equation (25).

Step 9: The sub-criteria weights were determined using Equation (26) and are presented in Table 6.

Steps 10-11: The overall performances of the alternatives were calculated using Equation (27). Ranks were given to the alternatives - from highest to lowest performance values. Results obtained from the 11 steps are given in Table 7.

Table 7: Ranking of alternatives corresponding to performance values for $p = 2$.

Candidate	Performance value, $J(A_i)$	Ranking
A_1	0.0054393	13
A_2	0.0045601	14
A_3	0.0031005	15
A_4	0.0082521	10
A_5	0.0127783	4
A_6	0.0088508	9
A_7	0.0133858	3
A_8	0.0126561	5
A_9	0.0077783	11
A_{10}	0.0183185	1
A_{11}	0.0104164	7
A_{12}	0.0126094	6
A_{13}	0.0077422	12
A_{14}	0.0175975	2
A_{15}	0.0100064	8

The ranking of the alternatives in descending order is:

$$A_{10} \succ A_{14} \succ A_7 \succ A_5 \succ A_8 \succ A_{12} \succ A_{11} \succ A_{15} \succ A_6 \succ A_4 \succ A_9 \succ A_{13} \succ A_1 \succ A_2 \succ A_3.$$

Hence, the optimal alternative is A_{10}.

5 Comparison and Discussions

In this study, the alternatives were also sorted and ranked using fuzzy TOPSIS and other kinds of fuzzy divergence measures. Ranks obtained by this study are then compared with various methods, as given in Table 8 below.

Table 8: Ranking order of alternatives using various methods.

Methods	Principal Measure	Ranking order
Fuzzy TOPSIS by Awasthi et al. [8]	Fuzzy sets and Euclidean distance	$A_{10} \succ A_{14} \succ A_{12} \succ A_5 \succ A_7 \succ A_8 \succ A_{11} \succ A_{15} \succ A_6 \succ A_4 \succ A_{13} \succ A_9 \succ A_1 \succ A_3 \succ A_2$
Fuzzy Divergence Measures by Rani et al. [41]	Fuzzy sets and divergence measure	$A_{14} \succ A_{10} \succ A_7 \succ A_5 \succ A_8 \succ A_{12} \succ A_{15} \succ A_{11} \succ A_4 \succ A_{13} \succ A_1 \succ A_6 \succ A_2 \succ A_9 \succ A_3$
Fuzzy Divergence Measures by Rani et al. [42]	Fuzzy sets, connective and divergence measure	$A_{10} \succ A_{14} \succ A_7 \succ A_5 \succ A_{12} \succ A_8 \succ A_{15} \succ A_{11} \succ A_4 \succ A_{13} \succ A_6 \succ A_9 \succ A_1 \succ A_2 \succ A_3$
Proposed Method	Fuzzy sets, connective and divergence measure	$A_{10} \succ A_{14} \succ A_7 \succ A_5 \succ A_8 \succ A_{12} \succ A_{11} \succ A_{15} \succ A_6 \succ A_4 \succ A_9 \succ A_{13} \succ A_1 \succ A_2 \succ A_3$

Table 9: Correlation coefficient values between alternative rankings computed using different MCDM methods.

	Fuzzy TOPSIS by Awasthi et al. [8]	Fuzzy Divergence Measures by [41]	Fuzzy Divergence Measures by [42]	Proposed Method
Fuzzy TOPSIS by Awasthi et al. [8]	1.0000			
Fuzzy Divergence Measures by Rani et al. [41]	0.9250	1.0000		
Fuzzy Divergence Measures by Rani et al. [42]	0.9679	0.9750	1.0000	
Proposed Method	0.9679	0.9429	0.9750	1.0000

Table 10: Average CPU time for MCDM methods.

Methods	Average CPU time, ms
Fuzzy Divergence Measures by Rani et al. [41]	134.375
Fuzzy Divergence Measures by Rani et al. [42]	128.125
Proposed Method	82.813

There is a slight difference in the ranking order of fuzzy TOPSIS, fuzzy divergence measures, and the proposed methods for staff performance appraisal. All methods except the fuzzy divergence measure proposed by Rani et al. [41] conclude that the optimal alternative A_{10} is the best staff in the department. The proposed method of divergence measure has several advantages. First, the criteria weights and alternatives are evaluated using linguistic variables, making the evaluation process easier for experts. We also used the GMIR method for the MCDM problem, which improved the precision of the results. Second, the proposed method employs a conventional theory of simultaneous fulfilment of the two concepts of the TOPSIS method, which aims to ensure that the result is farthest away from NIS, and at the same time, nearest to PIS. Third, a new divergence measure is proposed as it can evaluate any possible score value of alternatives, overcoming the deficiency of the existing measures and eliminating the abnormality of results. Finally, the proposed method used the aggregation connective measure, which determines the intersection of PIS and NIS for each score value. This is more reasonable than the existing methods that aim to assess the closeness coefficient of each alternative.

Based on the correlation coefficients between alternative rankings evaluated using various MCDM methods in Table 9, it is seen that there are strong relationships between the proposed method and the other MCDM methods. The differences in ranking for each alternative based on those methods are too small since all the correlation coefficient values are more than 0.9. Since the proposed method uses the idea of TOPSIS, we decided to compare the results with the method that uses the classical concept of TOPSIS presented by Awasthi et al. [8]. As a result, the alternative rankings of the proposed method are justifiable in terms of the TOPSIS idea. The results of proposed method are compared with the methods using divergence measure techniques. In this case, we compare the CPU time, the time a CPU was used for operational evaluation process, as in Table 10. Hence, it can be concluded that the proposed method is the best method as its CPU time is the lowest compared to the other two methods.

6 Conclusions

Performance evaluation is critical for the growth of any organisation or institution. Many decision-making approaches have been introduced to implement performance appraisal. Current research proposed a novel divergence measure coupled with the TOPSIS method to solve MCDM problems. Fuzzy linguistic variables are employed to measure the scores of alternatives and the importance of criteria since the assessment is always related to qualitative measurement. To verify the applicability of the suggested technique, an example is provided towards the end of this paper. A comparative study with other existing MCDM methods revealed a similar ranking for staff performance appraisal. At this point, the proposed method has a high potential for tackling issues related to MCDM in fuzzy sets, where the alternatives are measured using fuzzy values as criteria. This method could be improved by broadening our research to include the type of fuzzy intervals.

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A Appendix A

Table 11: Evaluations of candidates' performance against the sub-criteria by two experts.

		C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	C_{41}
A_1	E_1	α	γ	γ	γ	β	β	β	β	β	β	α	β	β	γ
	E_2	α	γ	γ	γ	β	β	β	β	β	β	α	β	β	γ
A_2	E_1	α	β	β	γ	β	β	β	γ	γ	γ	α	γ	β	γ
	E_2	α	β	β	γ	γ	β	β	γ	γ	γ	α	γ	β	γ
A_3	E_1	β	γ	γ	γ	γ	β	β	β	β	β	α	β	β	γ
	E_2	β	γ	γ	γ	γ	β	β	β	β	β	α	β	β	γ
A_4	E_1	α	γ	β	β	β	β	β	β	β	β	α	β	β	γ
	E_2	α	γ	β	β	β	β	β	β	β	β	α	β	β	γ
A_5	E_1	α	β	β	β	β	β	β	β	α	α	α	β	β	β
	E_2	α	β	β	β	β	β	β	β	α	α	α	β	β	β
A_6	E_1	β	β	β	β	β	β	β	β	β	β	α	β	β	γ
	E_2	β	β	β	β	β	β	β	β	β	β	α	β	β	γ
A_7	E_1	α	β	β	β	β	β	β	β	α	α	α	α	β	γ
	E_2	α	β	β	β	β	β	β	β	α	α	α	α	β	γ
A_8	E_1	α	β	β	β	β	β	β	β	α	β	α	β	α	β
	E_2	α	β	β	β	β	β	β	β	α	β	α	β	α	β
A_9	E_1	β	β	β	β	β	β	β	β	β	β	α	γ	γ	γ
	E_2	β	β	β	β	β	β	β	β	β	β	α	γ	γ	γ
A_{10}	E_1	α	β	β	β	β	α	α	α	β	α	α	α	α	β
	E_2	α	β	β	β	β	α	α	α	β	α	α	α	α	β
A_{11}	E_1	α	β	β	β	β	β	β	β	β	β	α	β	β	β
	E_2	α	β	β	β	β	β	β	β	β	β	α	β	β	β
A_{12}	E_1	α	β	β	β	β	α	β	β	β	α	α	β	β	β
	E_2	α	β	β	β	β	α	β	β	β	α	α	β	β	β
A_{13}	E_1	α	γ	β	β	β	β	β	β	β	β	α	β	γ	γ
	E_2	α	γ	β	β	β	β	β	β	β	β	α	β	γ	γ
A_{14}	E_1	α	β	β	β	β	β	β	α	α	α	α	α	α	β
	E_2	α	β	β	β	β	β	β	α	α	α	α	α	α	β
A_{15}	E_1	α	β	β	β	γ	β	β	β	α	β	α	β	β	β
	E_2	α	β	β	β	γ	β	β	β	α	β	α	β	β	β